Convexity of the effective action from functional flows

Daniel F. Litim ^{a,b}, Jan M. Pawlowski^c, and Lautaro Vergara^d

^{a} School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, U.K.
^{b} Physics Department, CERN, Theory Division, CH-1211 Geneva 23.
^{c} Institut für Theoretische Physik, University of Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany
^{d} Departamento de Física, Universidad de Santiago de Chile, Casilla 307, Santiago 2, Chile

We show that convexity of the effective action follows from its functional flow equation. Our analysis is based on a new, spectral representation. The results are relevant for the study of physical instabilities. We also derive constraints for convexity-preserving regulators within general truncation schemes including proper-time flows, and bounds for infrared anomalous dimensions of propagators.

Introduction.— Functional flows have been successfully used for perturbative as well as non-perturbative problems in quantum field theory and statistical physics [1, 2]. They provide a definition for finite generating functionals of the quantum theory, *i.e.* the effective action. The latter is a Legendre transform and therefore convex [3]. In general, convex effective actions admit stable solutions of the quantum equations of motions. In turn, non-convexities are linked to instabilities and have physical as well as technical origins. Physical instabilities range from those in condensed matter systems to QCD instabilities and are *e.g.* related to tunnelling phenomena and decay properties [1]. On the other hand instabilities may reflect artefacts of the underlying truncation or parameterisation. It is mandatory to properly distinguish between these two qualitatively different scenarios.

Functional flows for the effective action have been constructed from first principles as well as from a renormalisation group improvement. A large class of the latter are well-defined truncations of first-principle flows within a background field formulation [4, 5], including proper-time flows [6, 7, 8]. For first-principle flows, the set of convex functionals is an attractive fixed point of the full flow. Since truncations to the full problem at hand are inevitable, it is vital to identify convexity-preserving expansion schemes and regulators, and to determine limitations of widely used approximation schemes. In this Letter we provide a constructive proof of convexity for the effective action hence closing the present conceptual gap. Throughout, we illustrate our reasoning at the example of the derivative expansion.

Functional flows and spectral representation.— The analysis is done within a new, spectral representation for functional flows w.r.t. an infrared cutoff scale k,

$$\partial_t \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2} \int_{\mathbb{R}} d\lambda \, \rho(\bar{\phi}; \lambda) \langle \psi_\lambda | \frac{1}{\Gamma_k^{(2,0)} + R_k} \partial_t R_k | \psi_\lambda \rangle \tag{1}$$

and $t = \ln k$. Here, ϕ is the dynamical quantum field and $\bar{\phi}$ is some background configuration, *e.g.* the vacuum field. The flow (1) depends on the full propagator of the quantum field ϕ. The propagator is written in terms of the two-point function $\Gamma^{(2,0)}$ of ϕ. Gener-

ally we define mixed functional derivatives w.r.t. ϕ and $\bar{\phi}$ as $\Gamma_k^{(n,m)} = \delta^{n+m}\Gamma_k/(\delta\phi^n\delta\bar{\phi}^m)$ [4]. The regulator $R_k = R_k(\Gamma_k^{(2,0)}[\bar{\phi}, \bar{\phi}])$ depends on the two-point function evaluated at the background field $\bar{\phi}$, and the spectral values of $\Gamma_k^{(2,0)}$ are defined by

$$\lambda(\phi, \bar{\phi}) = \langle \psi_\lambda | \Gamma_k^{(2,0)}[\phi, \bar{\phi}] | \psi_\lambda \rangle \,, \tag{2}$$

with eigenfunctions ψ_λ, and $\rho(\bar{\phi}; \lambda)$ is the spectral density of $\lambda(\bar{\phi}, \bar{\phi})$. The flow (1) is fully equivalent to standard background field flows studied in [4]. We note that, since $\Gamma_k^{(2,0)}$ can have negative spectral values, $R_k > 0$ also has to be defined for negative arguments. In the absence of further scales we write the regulator as $R_k(\lambda) = \lambda \, r(\lambda/k^2)$ with k-independent function r. As an example, consider (1) for a scalar theory in the standard momentum representation to leading order in the derivative expansion. The spectral values are $\lambda(\bar{\phi}, \bar{\phi}) = p^2 + U_k^{(2,0)}[\bar{\phi}, \bar{\phi}]$, and the measure and the spectral density in d dimensions are $d\lambda \, \rho(\bar{\phi}; \lambda) = \frac{1}{2} dp^2 (p^2)^{d/2-1}/(2\pi)^{d/2}$ with density

$$\rho(\bar{\phi}, \lambda) = \frac{1}{2} \frac{1}{(2\pi)^{d/2}} (\lambda - U_k^{(2,0)})^{d/2-1} \theta[\lambda - U_k^{(2,0)}] \,, \tag{3}$$

where $U_k^{(2,0)} = U_k^{(2,0)}[\bar{\phi}, \bar{\phi}]$. Convexity is proven by showing that the spectral values for $\Gamma_k^{(2,0)} + R_k$ are positive for all k. To that end, we first study (1) within an additional approximation for the remaining matrix element. Then we extend the proof to the general case. For $\bar{\phi} = \phi$ the spectral representation simplifies

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \int_{\mathbb{R}} d\lambda \, \rho(\phi; \lambda) \, \frac{\partial_t R_k + \partial_t \lambda(\phi; \lambda) \partial_\lambda R_k}{\lambda + R_k(\lambda)} \,. \tag{4}$$

In (4) we have defined $\Gamma_k[\phi] = \Gamma_k[\phi, \phi]$, which only depends on one field. We have also used that

$$\langle \psi_\lambda | \partial_t \Gamma_k^{(2,0)} | \psi_\lambda \rangle = \partial_t \langle \psi_\lambda | \Gamma_k^{(2,0)} | \psi_\lambda \rangle = \partial_t \lambda(\phi; \lambda) \,, \tag{5}$$

for $\lambda \neq 0$ and $\partial_t \lambda(\phi; \lambda) = \partial_t \lambda(\phi, \phi; \lambda)$. In (5) we have used that $\langle \partial_t \psi_\lambda | \psi_\lambda \rangle = 0$ for normalised functions with $\langle \psi_\lambda | \psi_\lambda \rangle = 1$ and $\Gamma_k^{(2,0)}[\phi, \phi] | \psi_\lambda \rangle = \lambda | \psi_\lambda \rangle$. The simplicity

of the spectral flow (4) was payed for with the fact that it is not closed [4]: the field-dependent input on the rhs, ρ and λ require the knowledge of $\Gamma_k^{(2,0)}[\phi, \phi] \neq \Gamma_k^{(2)} = \Gamma_k^{(2,0)} + 2\Gamma_k^{(1,1)} + \Gamma_k^{(0,2)}$. Hence the simplicity of (4) can only be used with the approximation [9]

$$\Gamma_k^{(2)}[\phi] = \Gamma_k^{(2,0)}[\phi, \phi] \,. \tag{6}$$

Within this truncation (1) turns into a closed flow equation for $\Gamma_k[\phi]$. The spectral values are given by $\lambda(\phi) = \langle \psi_\lambda | \Gamma^{(2)}[\phi] | \psi_\lambda \rangle$. The flow (4) with (6) allows for the construction of gauge invariant flows [7, 9, 10].

If also neglecting the contributions in (4) that are proportional to $\partial_t \lambda$, we are led to the widely used proper-time flows, see [4], with spectral representation

$$\partial_t \Gamma_k[\phi] = \tfrac{1}{2} \int_{\mathbb{R}} d\lambda \, \rho(\phi; \lambda) \frac{\partial_t R_k(\lambda)}{\lambda + R_k(\lambda)} \,. \tag{7}$$

The only ϕ-dependence in (7) is that of $\rho(\phi; \lambda)$ as λ serves as an integration variable. In distinction to the full flow (1), we stress that convexity for the proper-time flow (7) is not automatically guaranteed by formal properties of the effective action. The flow (7) relies on the approximation (6), and $\Gamma_k[\phi]$ is not directly defined as a Legendre transform. Hence proving convexity for proper-time flows further sustains its nature as a well-controlled approximation of functional flows. Indeed, the representation (7) facilitates the analysis. Proving convexity from the flow itself is more difficult for the full flow, even though we know on general grounds that it entails convexity.

Convexity of proper-time flows.— If $\Gamma_k^{(2)}$ has negative spectral values they are bounded from below. Hence, the spectral density obeys $\rho(\phi; \lambda < \lambda_{\min}) \equiv 0$ for some finite $\lambda_{\min}$ for all ϕ. The flow $\partial_t \Gamma_k^{(2)}[\phi]$ entails the flow of the spectral values $\lambda(\phi)$ and, in particular, that of $\lambda_{\min}$. We shall prove that with $k \to 0$ the flow increases $\lambda_{\min}$, its final value being $\lambda_{\min}(k = 0) \geq 0$. The flow of λ is derived from (7) with (5) and (6). The field derivatives only hit ρ on the rhs of (7) and we arrive at

$$\partial_t \lambda(\phi) = \tfrac{1}{2} \int_{\mathbb{R}} d\lambda' \, \langle \rho^{(2)}(\phi; \lambda') \rangle_\lambda \frac{\partial_t R_k(\lambda')}{\lambda' + R_k(\lambda')} \,, \tag{8}$$

with $\langle \rho^{(2)} \rangle_\lambda = \langle \psi_\lambda | \rho^{(2)} | \psi_\lambda \rangle$. For the standard class of regulators used for proper-time flows [4], the flow reads

$$\partial_t \lambda(\phi) = \int_{\mathbb{R}} d\lambda' \, \langle \rho^{(2)}(\phi; \lambda') \rangle_\lambda \frac{1}{(1 + \lambda'/(mk^2))^m} \,. \tag{9}$$

Using (3), a simple example for $\rho^{(2)}$ is provided by the leading order derivative expansion in $d = 4$,

$$\langle \rho^{(2)} \rangle_\lambda = -\frac{1}{(8\pi^2)} \Big(U_k^{(4)} - 2(U_k^{(3)})^2 \partial_\lambda$$
$$- (\lambda - U_k^{(2)}) (U_k^{(3)})^2 \partial_\lambda^2 \Big) \theta[\lambda - U_k^{(2)}] \,. \tag{10}$$

We proceed by evaluating (8) for $\lambda_{\min}$. To that end we have to choose ϕ_0 that admit the spectral value $\lambda_{\min}$.

Note that the spectral density (and its derivatives) may vanish in more than two dimensions, $\rho(\phi_0; \lambda_{\min}) = 0$, *e.g.* in the above example of the derivative expansion with $\lambda_{\min} = U^{(2,0)}[\phi_0, \phi_0]$, see (3). Moreover, the proofs below work if no discrete set of low lying spectral values is present, such as come about in theories with non-trivial topology. However, it can be easily extended to this case as these modes can be separated due to their discreteness. Assume that $\lambda_{\min}$ stays negative in the limit $k \to 0$. Then, the propagator generically develops a singularity at the minimal spectral value at some cut-off scale k_{sing},

$$R_{k_{\text{sing}}}(\lambda_{\min}) = -\lambda_{\min} \,. \tag{11}$$

For example, (11) holds for (smooth) regulators with $R_{k=0} \equiv 0$. In (11) we have deduced from the parameterisation $R_k(\lambda) = \lambda \, r(\lambda/k^2)$ and continuity that the singularity is developed at λ_{sing}. Later we shall also discuss the general case. The contribution of the vicinity of the singularity dominates the integral if the singularity is strong enough. We use that $\rho(\phi; \lambda_{\min})$ and $\rho^{(2)}(\phi; \lambda_{\min})$ vanish for ϕ that do not admit the eigenvalue $\lambda_{\min}$. Consequently as operator equations we have

$$\rho^{(1)}(\phi; \lambda_{\min}) \equiv 0, \qquad \rho^{(2)}(\phi; \lambda_{\min}) \leq 0 \,, \tag{12}$$

in particular for $\phi = \phi_0$. The second identity follows within an expansion about ϕ_0 since the related term has to decrease the spectral density. With (12) the rhs of (8) is negative

$$\partial_t \lambda_{\min} \leq 0 \,, \tag{13}$$

and $\lambda_{\min}$ is increased for decreasing k. As long as Γ_k is differentiable w.r.t. ϕ this argument applies also for eigenvalues in the vicinity of $\lambda_{\min}$.

The condition (13) is necessary but not sufficient for convexity. A sufficient condition is given by the positivity of the gap $\epsilon = \lambda_{\min} + R_k(\lambda_{\min})$. Hence, for $\epsilon \to 0$ its flow $\partial_t \epsilon$ has to be negative. This leads to the constraint

$$\partial_t \lambda_{\min} \leq - \left. \frac{\partial_t R_k}{1 + \partial_\lambda R_k} \right|_{\lambda_{\min}} \,, \tag{14}$$

as $\partial_t R_k \geq 0$ implies $1 + \partial_\lambda R_k \geq 0$. At k_{sing} an upper bound for $\partial_t \lambda_{\min}$ is obtained from (8) with $\rho^{(2)} \propto (\lambda - \lambda_{\min})^{\alpha_\rho}$, where we count $\delta(x)$ as x^{-1}. The exponent is bounded from above, $\alpha_\rho \leq d/2 - 2$. This follows from the positivity of the anomalous dimension of the two point function, $\alpha > 0$ with $\lambda - \lambda_{\min} \propto p^{2(1+\alpha)}$, and $\rho \propto p^{2(d/2-1)}$. Negative α would entail a diverging $\partial_{p^2} \lambda_{\min}$ which can only be produced from a diverging flow $\partial_t \partial_{p^2} \lambda_{\min}|_{k_{\text{sing}}}$. However, for $\alpha < 0$ this flow is finite due to the suppression factor $\rho^{(2)}$ and $\alpha > 0$ follows, for all k. We expand the integrand in (8) about $\lambda_{\min}$ as

$$\frac{\partial_t R_k(\lambda)}{\lambda + R_k(\lambda)} = \frac{c_1}{\epsilon^\delta + c_2 (\lambda - \lambda_{\min})^\beta} + \text{sub-leading} \,. \tag{15}$$

with expansion coefficients c_1, c_2. The sub-leading terms comprise higher order terms in ϵ and in $(\lambda - \lambda_{\min})$.

The exponents $\delta(R_k), \beta(R_k)$ are regulator-dependent real positive numbers, and essential singularities are covered by the limit $\delta, \beta \to \infty$, e.g. (9) with $\beta = m \to \infty$. In the latter case the essential singularity is obtained at $k_{\mathrm{sing}} = 0$, and the regulator $R_{k=0}(\lambda) = -\lambda$ for $\lambda < 0$. We conclude that a sufficient growth of $\lambda_{\min}$ is guaranteed for $\beta \geq d/2-1$ which is identical with $m \geq d/2-1$ in (9). Lower m correspond to flows for $\Gamma^{(2)}$ with UV problems, in particular the Callan-Symanzik flow for $m = 1$ in $d \geq 4$, whereas the above constraint comes from an IR consideration: for the flows (9) UV finiteness of the flow and the demand of an IR singularity for the flow of $\lambda_{\min}$ are the same, as the flows are monomials in the propagator. For general regulators there is no UV-IR interrelation. For $\beta \geq d/2-1$ it follows from (8) that

$$\lim_{\epsilon \to 0} \partial_t \lambda_{\min} = -\infty \,, \tag{16}$$

satisfying (14) for $\partial_\lambda R(\lambda_{\min}) > -1$. In (16) we have used that for small enough ϵ the integral is dominated by the vicinity of the pole where $\langle \rho^{(2)}(\phi; \lambda) \rangle_{\lambda_{\min}} \leq 0$. For small enough ϵ the flow (16) exceeds the decrease of R_k, and the singularity cannot be reached. We conclude that $\lambda_{\min} + R_k(\lambda_{\min}) > 0$ and consequently

$$\lim_{k \to 0} \lambda_{\min} \geq 0 \,, \tag{17}$$

which entails convexity for proper-time flows. Let us also study the convexity of truncations to (7): the arguments above straightaway applies to truncations Γ_{trunc} which admit the direct use of the full field-dependent propagator $(1 + \Gamma_{\mathrm{trunc}}^{(2)}[\phi]/(m\,k^2))^{-1}$ in (7). If expansions $\Gamma_{\mathrm{trunc}} = \Gamma_1 + \Delta\Gamma$ are used on the rhs of (7) (leading to $(1 + \Gamma_1^{(2)}[\phi]/(m\,k^2))^{-1}$), convexity might become a difficult problem. Then, the arguments above entail convexity of Γ_1 for $k \to 0$ but not necessarily for Γ_{trunc}. We close with the remark that for $\beta < d/2-1$ convexity cannot be proven. Indeed it can be shown that then convexity is not guaranteed for $k = 0$ [13]. This holds true for full flows within lowest order derivative expansion [14]. In the latter case it hints at inappropriate initial conditions.

For regulators that do not lead to singularities (11) in the propagator necessarily $R_{k=0}(\lambda) > |\lambda|$ for $\lambda < 0$, and convexity of $\Gamma_{k=0}$ cannot be guaranteed. Note also, that for non-convex effective action we keep an explicit regulator dependence for $k = 0$.

Convexity of full flows and general theories.— The flow on $\partial_t \lambda$ is given by the second derivative w.r.t. ϕ of (1) at $\bar{\phi} = \phi$. We evaluate the flow at $\phi = \phi_0$ with minimal spectral value $\lambda_{\min}(\phi_0)$, and in the vicinity of the singularity, $\lambda_{\min} + R_k(\lambda_{\min}) = \epsilon$. We are led to

$$\partial_t \lambda_{\min} = \tfrac{1}{2} \int_{\mathbb{R}} d\lambda' \, \rho(\phi_0; \lambda')$$

$$\times \langle \, \langle \psi_{\lambda'} | \left(\frac{1}{\Gamma_k^{(2,0)} + R_k} \right)^{(2,0)} | \psi_{\lambda'} \rangle \, \rangle_{\lambda_{\min}}$$

$$\times \left[\partial_t R_k(\lambda') + \partial_t \lambda(\phi_0; \lambda') \partial_{\lambda'} R_k(\lambda') \right] + \Delta \,, \tag{18}$$

where Δ comprises sub-leading terms that are proportional to off-diagonal matrix elements of the propagator. For $\beta \geq d/2 - 1$ these terms are suppressed by higher order in ϵ. There are no terms proportional to $\rho^{(2)}$ and $\partial_t \langle \lambda(\phi)^{(2)} \rangle_{\lambda_{\min}}$ as ρ, $\partial_t \lambda$ and $|\psi_\lambda\rangle$ only depend on $\bar{\phi}$. All terms in (18) are proportional to the diagonal matrix elements in the second line. Similarly as for $\rho^{(2)}$ it also follows that the relevant diagonal matrix element in the integral in (18) is negative in the vicinity of $\lambda_{\min}$: the propagator takes its maximal spectral value at ϕ_0 and hence its second field derivative at ϕ_0 is negative. We conclude for $\beta \geq d/2 - 1$ that (18) is only solved for $\partial_t R_k + \partial_t \lambda \, \partial_\lambda R_k \to 0$ for $\lambda \to \lambda_{\min}$. This entails that $\partial_t \epsilon = \partial_t \lambda_{\min} +$ sub-leading, and leads to

$$\partial_t \epsilon = -\left. \frac{\partial_t R_k}{\partial_\lambda R_k} \right|_{\lambda_{\min}} + \text{sub-leading} \,. \tag{19}$$

The flow of the gap ϵ has to be negative for $\epsilon \to 0$ in order to ensure convexity. This leads to the constraint

$$\left. \frac{\partial_t R_k}{\partial_\lambda R_k} \right|_{\lambda_{\min}} \geq 0 \,, \tag{20}$$

for small enough ϵ. We remark that (20) cannot hold for all λ as R_k has to decay for large positive λ, and has to vanish for $k \to 0$. Furthermore the above proof at $\phi = \bar{\phi}$ is sufficient for convexity for all ϕ. If evaluating the full flow at some $\bar{\phi} \neq \phi_0$ the spectral density is non-vanishing at this $\bar{\phi}$ and we get convexity for $\beta \geq 1$. This completes the convexity proof of general flows.

The proof is straightforwardly extended to theories with general field content with fields ϕ_i, $i = 1, ..., N$. For illustration we restrict ourselves to regulators that are diagonal in field space with entries $R_k^{(1)}, ..., R_k^{(N)}$ and arguments $\Gamma_{k,ii}^{(2,0)}$, the diagonal elements of the two-point function. Choosing a spectral representation in terms of the eigenfunctions $\psi_\lambda^{(i)}$ and spectral values $\lambda^{(i)}$ of $\Gamma_{k,ii}^{(2,0)}$, the integrand in (1) reads

$$\sum_{i=1}^{N} \rho_i(\bar{\phi}; \lambda) \langle \psi_\lambda^{(i)} | \left(\frac{1}{\Gamma_k^{(2,0)} + R_k} \right)_{ii} \partial_t R_k^{(i)} | \psi_\lambda^{(i)} \rangle \,. \tag{21}$$

Note that the spectral values $\lambda^{(i)}$ are in general not spectral values of $\Gamma_k^{(2)}$. However, singularities of diagonal elements of the propagator in (21) are in one to one correspondence to vanishing spectral values of diagonal elements of the two point function, $\Gamma_{k,ii}^{(2,0)} + R_k^{(i)}$. Hence, $\lambda^{(i)} \geq 0$ at $k = 0$ for all i follows directly from the proof for theories with only one field, and it entails $\lambda \geq 0$, where λ are the spectral values of $\Gamma^{(2,0)}$ at $k = 0$.

Derivative expansion.— To illustrate our findings, we consider the infrared running of the scale-dependent effective potential $U_k(\phi)$ in $d = 3$ dimensions for a N-component real scalar field ϕ^a in the large-N limit, to leading order in a derivative expansion, e.g. [15]. Here, the running potential U_k is obtained from integrating
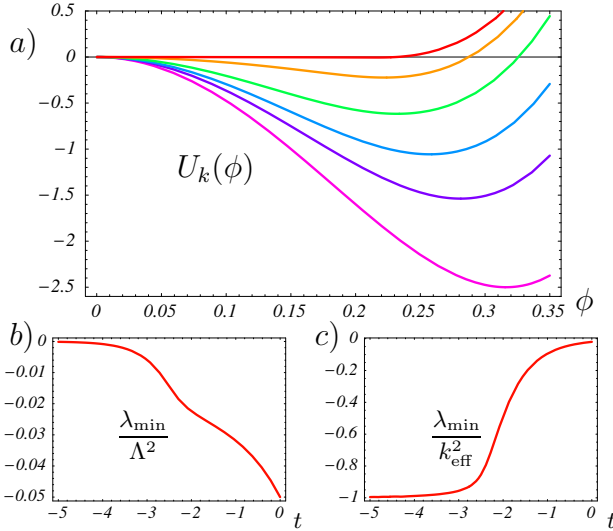
FIG. 1: Approach to convexity in terms of a) the effective potential (rescaled) for $t = \ln k/\Lambda = 0, -0.5, -1, -2, -3, -5$ from bottom to top, and b, c) the lowest spectral value $\lambda_{\min}$; $d = 3$, $\mu^2/\Lambda^2 = -0.05$, $g/\Lambda = 1$ (see text).

the proper-time flow (7) in the parametrisation (9) with $m = d/2 + 1$, see Fig. 1$a) - c$). This value of m corresponds to an optimised flow [11, 12], similar plots follow for all $m \geq 3/2$ [13]. The boundary condition is $U_\Lambda = \frac{1}{2}\mu^2\phi^2 + \frac{1}{8}g\phi^4$ at $k = \Lambda$. For $\mu^2/g < 0$, the potential U_Λ displays spontaneous symmetry breaking with a global minimum at $\phi^2_{\min,\Lambda} = -2\mu^2/g$. With decreasing k, the minimum runs towards smaller values, settling at $\phi_{\min,0} < \phi_{\min,\Lambda}$, see Fig. 1$a$). For fields in the non-convex regime of the potential the flow displays negative spectral values, corresponding to an instability. Here, the lowest spectral value is given by the running mass term at vanishing field, $\lambda_{\min} = U''_k(0) \leq 0$, which smoothly tends to zero for $k \to 0$, see Fig. 1b). Once $\phi_{\min}$ has settled, the

running of $\lambda_{\min}$ changes qualitatively: in the infrared, the size of the spectral value is set by the effective cutoff scale $k^2_{\mathrm{eff}}(k) = mk^2$, see Fig. 1$c$), and the entire inner part of the potential becomes convex.

Discussion. — We have provided a proof of convexity for general functional flows (1), subject to simple constraints on the set of regulators. The constraints are $\beta \geq d/2 - 1$ derived from (15), as well as (20) for full flows. The finiteness of $\partial_t\lambda_{\min}$ at the singularity and (20) seemingly indicates worse convexity properties for the full flow (1) (at $\phi = \phi_0$) in comparison with proper-time flows. However, full flows entail convexity by definition. This paradox is resolved by considering the initial condition. Only consistent choices correspond to a path integral and lead to convex effective actions at $k = 0$. Hence, regulators that violate (20) can be used to test the consistency of initial conditions for Γ_k for full flows. This allows us to investigate physical instabilities within these settings.

In addition we have proven positivity of the infrared anomalous dimension of the propagator, $\alpha \geq 0$. Negative α require additional fields with at least one strictly positive anomalous dimension. The latter scenario is relevant *e.g.* for Landau gauge QCD, where [16] already anticipates the general result.

The present work also finalises the analysis initiated in [4, 5], and fully establishes proper-time flows as well-defined, convexity-preserving approximations of first-principle flows. Note that in the proper-time approximation the standard regulators leading to (9) violate (20). For stable flows beyond (7) one should modify these regulators for negative spectral values.

[1] J. Berges, N. Tetradis and C. Wetterich, Phys. Rept. **363** (2002) 223; J. Polonyi, Central Eur. J. Phys. **1** (2004) 1; M. Salmhofer and C. Honerkamp, Prog. Theor. Phys. **105** (2001) 1.

[2] D. F. Litim and J. M. Pawlowski, in *The Exact RG*, Eds. Krasnitz et al, World Sci (1999) 168.

[3] L. O'Raifeartaigh, A. Wipf and H. Yoneyama, Nucl. Phys. B **271** (1986) 653.

[4] D. F. Litim and J. M. Pawlowski, Phys. Rev. D **66** (2002) 025030; Phys. Lett. B **546** (2002) 279.

[5] D. F. Litim and J. M. Pawlowski, Phys. Lett. B **516** (2001) 197, Phys. Rev. D **65** (2002) 081701.

[6] S.-B. Liao, Phys. Rev. D **53** (1996) 2020.

[7] S.-B. Liao, Phys. Rev. D **56** (1997) 5008.

[8] B.J. Schäfer, H.J. Pirner, Nucl. Phys. A **627** (1997) 481; Nucl. Phys. A **660** (1999) 439; G. Papp, B.J. Schäfer, H.J. Pirner, J. Wambach, Phys. Rev. D **61** (2000) 096002; A. Bonanno and D. Zappala, Phys. Lett. B **504** (2001) 181. A. Bonanno and G. Lacagnina, Nucl. Phys. B **693** (2004) 36.

[9] J. M. Pawlowski, Int. J. Mod. Phys. A **16** (2001) 2105; Acta Phys. Slov. **52** (2002) 475; D. F. Litim and J. M. Pawlowski, JHEP **0209** (2002) 049.

[10] M. Reuter and C. Wetterich, Nucl. Phys. B **417** (1994) 181; F. Freire, D. F. Litim and J. M. Pawlowski, Phys. Lett. B **495** (2000) 256; H. Gies, Phys. Rev. D **66** (2002) 025006.

[11] D. F. Litim, Phys. Rev. D **64** (2001) 105007; Phys. Lett. B **486** (2000) 92; Nucl. Phys. B **631** (2002) 128.

[12] J. M. Pawlowski, hep-th/0512261.

[13] D. F. Litim, J. M. Pawlowski, L. Vergara, in preparation.

[14] D. F. Litim, in preparation.

[15] N. Tetradis and C. Wetterich, Nucl. Phys. B **383** (1992) 197; D. F. Litim and N. Tetradis, hep-th/9501042; Nucl. Phys. B **464** (1996) 492.

[16] J. M. Pawlowski, D. F. Litim, S. Nedelko and L. von Smekal, Phys. Rev. Lett. **93** (2004) 152002.